A GAUGE INVARIANT UNITARY THEORY FOR PION PHOTOPRODUCTION [1]

C. H. M. van Antwerpen and I. R. Afnan
School of Physical Sciences, The Flinders University of South Australia,
Bedford Park, South Australia 5042, Australia

ABSTRACT: A covariant, unitary and gauge invariant theory for pion photoproduction on a single nucleon is presented. To achieve gauge invariance at the operator level one needs to include both the πN and $\gamma\pi N$ thresholds. The final amplitude can be written in terms of a distorted wave in the final πN channel provided one includes additional diagrams to the standard Born term in which the photon is coupled to the final state pion and nucleon. These additional diagrams are required in order to satisfy gauge invariance.

Most calculations to date for pion photoproduction on a single nucleon have included two ingredients: (i) A Born term which is taken to be gauge invariant. (ii) A final state interaction or distortion that is needed to satisfy the Watson theorem or unitarity. However, the inclusion of the pionic degrees of freedom into the problem changes the current and charge distribution and thus the coupling of the photon to the nucleon. Hence any consistent theory of pion photoproduction has to include the effect of the pionic degrees of freedom on the charge and current distribution. In other words one needs to satisfy unitarity and gauge invariance at the same time. Clearly, the inclusion of both of these symmetries to all orders requires a full solution to the field theory. Here, we

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propose to present a formulation that satisfies two-body unitarity and gauge invariance to first order in the electromagnetic coupling.

The starting point of this formulation is the one particle irreducible πNN three-point Green's function given by

$$\begin{aligned} G(q,p';p) &= \int d^4x_1 d^4x_2 d^4x_3\, e^{i(p'\cdot x_3+q\cdot x_1-p\cdot x_2)} < 0|T(\phi(x_1)\psi(x_2)\bar{\psi}(x_3))|0 > \\ &= S(p')\,\Delta(q)\,\Lambda_5^{(1)\dagger}(q,p';p)\,S(p)\ , \end{aligned} \tag{1}$$

where the spin, isospin labels have been suppressed. Here, S is the nucleon propagator, Δ is the pion propagator, and $\Lambda_5^{(1)\dagger}$ is the one pion irreducible πNN vertex. Since at this stage we are considering two-body unitarity only, we will restrict the dressing of the nucleon so that the final $\pi N \leftarrow \gamma N$ amplitude has at most one pion in every intermediate state. This allows us to write the πNN three-point Green's function as

$$G(q,p';p) = S_0(p')\,\Delta(q)\,\Lambda_5^{(1)\dagger}(q,p';p)\,S(p)\ , \tag{2}$$

where $S_0(p) = (\not{p}-m)^{-1}$ and $S(p) = (\not{p}-m-\Sigma^{(1)}(\not{p}))^{-1}$, with $\Sigma^{(1)}$ including all contributions to two-body unitarity from mass renormalization.

The photoproduction amplitude is then constructed by gauging Eq. (2) and applying the LSZ reduction to the resulting four-point Green's function. The method of gauging employed was the minimal substitution $p_\mu \rightarrow p_\mu - eA_\mu$, where p_μ is the four momentum of the particle. This results in the following substitutions [1, 2]

$$S(\hat{p}) \;\rightarrow\; S(\hat{p}) + S(p')\Gamma_\mu(k,p',p)S(p)A^\mu \quad , \tag{3}$$

$$\Delta(\hat{q}) \;\rightarrow\; \Delta(\hat{q}) + \Delta(q')\Gamma^\pi_\mu(k,q',q)\Delta(q)A^\mu \quad , \tag{4}$$

$$\Lambda_5^{(1)\dagger}(\hat{q},\hat{p}',\hat{p}) \;\rightarrow\; \Lambda_5^{(1)\dagger}(\hat{q},\hat{p}',\hat{p}) + \Gamma^{CT}_\mu(k,q,p',p)A^\mu \quad . \tag{5}$$

i.e. the photon couples to all possible external lines and vertices present. We have followed a procedure developed by Ohta [2] in which (i) a Taylor series expansion is assumed to exist for the form factors present, (ii) perturbation theory can be used to replace various operators by eigenvalues, and (iii) we restrict ourselves to first order in the electromagnetic coupling. Applying the substitutions of Eqs. (3-5) to Eq. (2) leads to the four classes of diagrams [3] describing the photoproduction amplitude,

$$\Lambda_5^{(1)\dagger}\,S\,\Gamma^{(1)}_\mu + \Gamma^{(2)}_\mu\,S_0\,\Lambda_5^{(1)\dagger} + \Gamma^{\pi(2)}_\mu\,\Delta\,\Lambda_5^{(1)\dagger} + \Gamma^{CT(1)}_\mu\ , \tag{6}$$

where $\Gamma^{(1)}_\mu$ is the one-particle irreducible photon nucleon vertex, $\Gamma^{\pi(2)}_\mu$ is the photon pion vertex which is taken to be the bare vertex, and $\Gamma^{CT(1)}_\mu$ is the $\pi N \leftarrow \gamma N$ amplitude resulting from the coupling of the photon to the πNN vertex, $\Lambda_5^{(1)\dagger}$. Here the irreducibility

is given with respect to the number of pions and nucleons only. This amplitude is by definition gauge invariant with the photon vertices satisfying their corresponding Ward-Takahashi identities [1, 4, 5].

To establish the fact that the amplitude for $\pi N \leftarrow \gamma N$, as given in Eq. (6) satisfies two-body unitarity, we follow the procedure of Araki and Afnan (AA) [6] and classify a diagram's contribution to the amplitude according to it's irreducibility. However, unlike AA who included first the πN threshold for two-body unitarity and then the $\pi\pi N$ and $\gamma\pi N$ cuts for three-body unitarity, in this case we include both the πN and $\gamma\pi N$ unitarity cuts, since the corresponding branch points are at the same energy, to satisfy two-body unitarity.

Exposing the πN unitarity cut results in the following integral equation for the πN amplitude [6]

$$t^{(0)} = v(1 + g t^{(0)}) \tag{7}$$

where v is the Born amplitude given by $v = t^{(2)} + \Lambda_5^{(2)\dagger} S_0 \Lambda_5^{(2)}$, and $g = S_0\,\Delta$ is the πN propagator. For the $\pi N \leftarrow \gamma N$ amplitude exposing the πN cut gives

$$M^{(0)} = \tilde{v} + v g M^{(0)} \quad , \tag{8}$$

with $\tilde{v} = M^{(2)} + \Lambda_5^{(2)\dagger} S_0 \Gamma^{(2)}$. In this case $\tilde{v}$ is not the Born amplitude, since exposing the $\gamma\pi N$ cut in $\Gamma^{(2)}$ gives

$$\Gamma^{(2)} = \Gamma^{(3)} + \Gamma^{3(3)} g \Lambda_5^{(1)\dagger} \quad , \tag{9}$$

where $\Gamma^{3(3)}$ is the $N \leftarrow \gamma\pi N$ amplitude. Similarly, exposing the $\gamma\pi N$ cut in $M^{(2)}$ gives

$$M^{(2)} = \Gamma^{CT(3)} + \tilde{F}^{(3)}_{3;c} g \Lambda_5^{(1)\dagger} + \Gamma^{\pi(2)} \Delta \Lambda_5^{(1)\dagger} + \Gamma^{(3)} S_0 \Lambda_5^{(1)\dagger} \quad , \tag{10}$$

where $\Gamma^{CT(3)}$ is the $\pi N \leftarrow \gamma N$ amplitude, $\tilde{F}^{(3)}_{3;c}$ is the $N\pi \leftarrow \gamma\pi N$ amplitude, and $\Gamma^{\pi(2)}$ is the $\pi \leftarrow \gamma\pi$ amplitude. This results in $\tilde{v}$ being given by

$$\tilde{v} = \Gamma^{CT(3)} + \tilde{F}^{(3)}_{3;c} g \Lambda_5^{(1)\dagger} + \Gamma^{\pi(2)} \Delta \Lambda_5^{(1)\dagger} + \Gamma^{(3)} S_0 \Lambda_5^{(1)\dagger} + \Lambda_5^{(2)\dagger} S_0 \Gamma^{(3)} + \Lambda_5^{(2)\dagger} S_0 \Gamma^{3(3)} g \Lambda_5^{(1)\dagger} \tag{11}$$

which contains more physics than just the Born amplitude. By comparing the above result for the photoproduction amplitude which satisfies two-body unitarity, with the amplitude in Eq. (6), we can establish that they are in fact identical [7].

If we now iterate Eq. (8) and make use of Eq. (7), we find that

$$M^{(0)} = (t^{(0)} g + 1)\tilde{v} = (t^{(0)} g + 1)\tilde{v}_B + (t^{(0)} g + 1)\tilde{v}_R \quad , \tag{12}$$

Figure 1: These are the non-Born diagrams which contribute to the pion photoproduction amplitude. The numbers in the circle give the irreduciblity of each amplitude.

where $\tilde{v}_B$ is the Born amplitude and $\tilde{v}_R$ contains the additional diagrams required to maintain gauge invariance which are illustrated in Fig. 1.

This result also proves that the commonly used procedure for unitarizing the Born amplitude in terms of the πN amplitude, i.e.

$$M^{(0)} = (t^{(0)}\, g + 1)\, \tilde{v}_B \quad , \tag{13}$$

does not satisfy gauge invariance. This is due to the fact that the derivation of Eq. (13) involves the inclusion of the πN threshold, which is what unitarity requires, and only the lowest order diagrams containing a $\gamma\pi N$ intermediate state, into $\tilde{v}_B$. However, to satisfy gauge invariance, the full $\gamma\pi N$ threshold should also be included, which is what one expects considering the fact that the two thresholds start at the same energy. We should also note that the additional terms resulting from the inclusion of the $\gamma\pi N$ threshold give rise to the dressing of the vertices in $\tilde{v}_B$, and this dressing is required to satisfy gauge invariance.

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